EXPONENTIATION AT PARTONIC THRESHOLD FOR THE DRELL-YAN CROSS SECTION

T.O. Eynck[a], E. Laenen[a,b] and L. Magnea[c]
[a] NIKHEF Theory Group
Kruislaan 409, 1098 SJ Amsterdam, The Netherlands
[b] Institute for Theoretical Physics, Utrecht University
Leuvelaan 4, 3584 CE Utrecht, The Netherlands
[c] Dipartimento di Fisica Teorica, Università di Torino
and INFN, Sezione di Torino
Via P. Giuria 1, I-10125, Torino, Italy

The techniques leading to the resummation of threshold logarithms in the Drell-Yan cross section and other processes can be used to show that also terms independent on the Mellin variable N exponentiate. Comparison with explicit two-loop calculations shows that within this class of terms the exponentiation of the one-loop result together with the running of the coupling is the dominant effect at two loops.

1 Introduction

Threshold resummations [1] have been an active field of study in QCD for over two decades. They have a direct relevance to phenomenology [2], since they extend the range of applicability of perturbative methods for the computation of hard cross sections, but they are also interesting from a purely theoretical point of view, since they provide a method to probe the interface of perturbative and nonperturbative physics.

The central issue which is addressed by threshold resummations is the physics of inhibited radiation. At high energy, in a production process which forces the radiation of gauge bosons in the final state to be either soft or collinear to the hard partons carrying the bulk of the invariant mass, the validity of the perturbative expansion is jeopardized by the presence of large double (Sudakov) logarithms to all orders. The logarithms are the remainders of soft and collinear singularities, after the cancellations required by the mandatory infrared safety of the observable; they are large, because such a process has two

disparate scales: the hard scale s and the scale of soft radiation τs ($\tau \ll 1$); they can be exponentiated and resummed, thanks to the factorizability and universality properties of soft and collinear radiation.

To account for the constraint imposed by momentum conservation, the exponentiation of soft contributions takes place after a Mellin (or Laplace) transform with respect to the variable vanishing at threshold, τ. In terms of the Mellin variable N, conjugate to τ, one finds a hierarchy of contributions, with an increasingly mild behavior at large N, corresponding to a decreasing level of infrared sensitivity at small τ. Sudakov logarithms are the only terms growing with N, as $\alpha_s^n \log^k N$, with $k \leq 2n$. Then one finds terms independent of N, then terms suppressed by powers of N, which however may still have logarithmic behavior in N, such as $\alpha_s^n (\log^k N)/N$, with $k \leq 2n - 1$.

Most of the work done in the past on the subject of threshold resummations has focused on Sudakov logarithms, which have been shown to exponentiate to all logarithmic accuracies, and are currently being explicitly evaluated at NNL level [3]. It was clear from the beginning [4], however, that at least some of the N-independent terms arising in the relatively simple case of the Drell-Yan process were both numerically important and exponentiating together with the logarithms. Furthermore, at least a subset of the logarithmic terms suppressed by a power of N can also be resummed by similar techniques, and it has been shown that these terms have a considerable impact on cross sections of phenomenological interest [5]. It is therefore of both practical and theoretical interest to study the possibility of extending currently available resummation techniques beyond the case of Sudakov logs. Here, we will briefly discuss the exponentiation of N-independent terms in the relatively simple cases of the DIS structure function F_2 and of the Drell-Yan cross section, following [6].

2 Factorization and exponentiation for N-independent terms

The resummation of threshold logarithms can be derived by factorizing the relevant cross section to isolate soft and collinear enhancements. For the partonic Drell-Yan cross section this factorization takes the form [1]

$$\omega(N, \epsilon) = |H_{\rm DY}|^2 \, \psi(N, \epsilon)^2 \, U(N) + \mathcal{O}(1/N) \, , \tag{1}$$

where $\psi(N,\epsilon)$ is a parton distribution containing singular collinear contributions, while $U(N)$ is an eikonal cross section responsible for coherent soft radiation. The DIS structure function F_2 obeys a similar factorization

$$F_2(N,\epsilon) = |H_{\rm DIS}|^2 \, \chi(N,\epsilon) \, V(N) \, J(N) + \mathcal{O}(1/N) \, , \tag{2}$$

where χ and V play the same role as ψ and U, respectively, while $J(N)$ is a jet function summarizing the effect of collinear final state radiation.

These factorizations are valid up to corrections suppressed by powers of N, and thus they include N-independent terms. The key observation is that it is possible to separate, within each function, real emission diagrams from purely virtual contributions, and then to express all virtual (an thus N-independent) functions in terms of the quark form factor. Specifically one finds that

$$\begin{aligned} \omega(N,\epsilon) &= |\Gamma(Q^2,\epsilon)|^2 \, \psi_R(N,\epsilon)^2 \, U_R(N,\epsilon) + \mathcal{O}(1/N) \ , & (3) \\ F_2(N,\epsilon) &= |\Gamma(-Q^2,\epsilon)|^2 \, \chi_R(N,\epsilon) \, V_R(N,\epsilon) \, J_R(N,\epsilon) + \mathcal{O}(1/N) \, . & (4) \end{aligned}$$

Each factor in Eqs. (3) and (4) is now a pure exponential: for real emission contributions this was established in Ref. [1], while for the form factor in dimensional regularization it was proven in Ref. [7]. Taking the ratio of (3) and (4) one finds an exponentiated expression for the factorized DIS-scheme Drell-Yan cross section, valid up to corrections suppressed by powers of N, which can be organized in the form

$$\widehat{\omega}_{\rm DIS}(N) = \left| \frac{\Gamma(Q^2,\epsilon)}{\Gamma(-Q^2,\epsilon)} \right|^2 \exp \left[F_{\rm DIS}(\alpha_s) \right] \ \exp \left[\int_0^1 dz \, \frac{z^{N-1} - 1}{1 - z} \right. \tag{5}$$
$$\left. \left\{ 2 \int_{(1-z)Q^2}^{(1-z)^2 Q^2} \frac{d\xi^2}{\xi^2} \, A \left(\alpha_s(\xi^2) \right) - 2 B \left(\alpha_s \left((1-z) Q^2 \right) \right) + D \left(\alpha_s \left((1-z)^2 Q^2 \right) \right) \right\} \right] .$$

Explicit expressions for the various functions involved are listed in [6].

It is not difficult to generalize this result to the $\overline{\rm MS}$ scheme. The Mellin transform of the $\overline{\rm MS}$ quark distribution, $\phi(N,\epsilon)$, can, in fact, be written in exponential form, up to corrections suppressed by powers of N. Further, as shown in [6], it is possible to factorize real and virtual contributions to $\phi(N,\epsilon)$, so that virtual poles precisely cancel those arising from the quark form factor. The factorized $\overline{\rm MS}$-scheme Drell-Yan cross section can then be written as the

product of two finite exponential factors, one associated with real gluon emission, and one with purely virtual graphs. The final result can be organized in the form

$$\widehat{\omega}_{\overline{\rm MS}}\,(N) = \left|\frac{\Gamma(Q^2,\epsilon)}{\Gamma(-Q^2,\epsilon)}\right|^2 \left(\frac{\Gamma(-Q^2,\epsilon)}{\phi_V(Q^2,\epsilon)}\right)^2 \exp\left[F_{\overline{\rm MS}}\,(\alpha_s)\right]\ \exp\left[\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\right.$$
$$\left.\left\{2\int_{Q^2}^{(1-z)^2Q^2}\frac{d\mu^2}{\mu^2}\,A\left(\alpha_s(\mu^2)\right) + D\left(\alpha_s\left((1-z)^2Q^2\right)\right)\right\}\right]\,. \qquad (6)$$

Again, explicit expressions for the functions involved are given in Ref. [6]. Eqs. (5) and (6) are our main results: they generalize currently available resummations for the Drell-Yan partonic cross section to include the exponentiation of all N-independent terms. Clearly, the same reasoning and a similar formula apply for the partonic DIS structure function F_2 factorized in the $\overline{\rm MS}$ scheme.

3 Usage and impact of the exponentiation

It should be clear that the exponentiation of N-independent terms does not have the same predictive power as the resummation of Sudakov logarithms. In fact, for example, a one-loop calculation suffices to determine exactly the coefficients of the leading Sudakov logs to all orders, whereas constant terms, although exponentiating, receive nontrivial contributions order by order. The resummation formulas in Eqs. (5) and (6) can however be used in practice in at least two different ways.

First of all, for the purpose of analytic calculations, on can make use of the fact that all functions appearing in the factorizations (1) and (2) have precise operator definitions, as well as of the fact that in the resummations (5) and (6) real and virtual contributions are separately finite. These two facts provide alternative, and often simpler, methods to perform and test analytic calculations at the resummed level, without having to compute the full cross section at the relevant perturbative order. As an example, in Ref. [6] we were able to compute the second order perturbative coefficient of the function D, using only information arising from virtual graphs, and reproducing the result previously obtained [3, 8] by matching to the exact two-loop cross section.

From the point of view of phenomenology, although exponentiation of the one-loop result does not suffice to determine exactly any specific perturbative

coefficient at higher orders, it remains true that a nontrivial subset of higher order corrections originate from exponentiation. This subset provides a well-founded estimate of the uncertainty of the perturbative calculation due to unknown higher order corrections. To test this fact, in Ref. [6], we estimated the numerical values of N-independent terms at two loops, by using only one-loop information in the exponent, together with renormalization group running. We find that exponentiation predicts three quarters of the exact answer for these terms in the DIS scheme, whereas for the $\overline{\mathrm{MS}}$ scheme the prediction exceeds the exact value by about 70%. In both cases, we conclude that exponentiation gives a fairly reliable estimate of the complete calculation.

We conclude by noting that these results may be seen as a first step towards further generalizations of soft gluon resummation: first of all, it would be interesting to study the exponentiation of N-independent terms for more general QCD cross sections, in the presence of nontrivial color exchange; furthermore, a precise formulation of the exponentiation of Sudakov logarithms suppressed by a power of N would be of considerable phenomenological interest.

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